Transcript

In this problem, we are raising the disco ball with the following mechanism. We're assuming there is no slip between A, B and C. And we're asked to find what is the input angular velocity omega a, to give a certain speed of this disco ball. So we're also given all the dimensions of the three of the mechanism and the final speed. So we're going to start with that, we need the velocity, final velocity vf to be equal to two meters per second. And so that will be equal to the velocity at this point being equal to VF. Obviously, this is going to spin this mechanism. And each of these three components is going to have a different angular velocity. And we are asked to determine this angular velocity over here, which is omega A. We have omega C going this way. And we have omega B, going this way. So I'm going to solve this equation or this problem with vectors. But once you know how to solve it with vectors, you can really simplify it and solve it with scalars. But just to understand what is going on, we're going to use the vector equations. So since we know we're given everything on this side, we're going to start with this. And we're going to work our way across and determine the angular velocity of the input of the motor. So starting from this wheel over here, we know the velocity at a certain point, we know where the wheel is pinned, so we can determine the angular velocity. And the equation is as follows. We know the equation the velocity VF, and this is going to be equal to the angular velocity of C cross the radius of the point where this velocity is acting, which we will call m with respect to C. So we'll call this here. The radius in this is going to be R of M with respect to C. So we can solve this equation because we know this velocity here, and we know this radius, so we can fully solve for this angular velocity. And if we do that, we get the following. Omega c is equal to two meters per second, divided by 0.4 meters. And this is going to be in the negative k hat direction. So this is going to be equal to negative five radians per second in the k hat direction, and this is going to be equal to omega c. And the magnitude of omega c is going to be equal to five radians per second. And we're getting rid of that direction. So now we have determined omega c. And again, the way you do this, this is kind of a backwards cross product where we're not really solving the cross, but we were solving the cross product, and then we're equating the two components to VF over here. So VF only has a component in the vertical direction, so in the negative y direction, and so we just take the component in the negative, in the y direction, have this cross product and equate it to the F, all the other components obviously have to be zero. So you can see that omega will be in the k hat direction. And, and the magnitude is just this, the velocity divided by the radius. And we have now determined the angular velocity of C. Now, given the angular velocity at C, we can find the velocity at this point over here. And the velocity at this point over here will have the same magnitude as the velocity over here on the right, because they have the same radius. They're the same distance from the center from the location where this wheel is pinned, but the direction will be different the direction in this case points upwards, whereas on the right it points downwards. And this is again critical in solving this problem. So let's go ahead and solve for the interface. solve for the velocity at the interface between C and B. So we'll call I'll just point Pone. And we'll call the point between B and A Ptwo. And we'll call the radius between the center and the pin, or the center where there's no velocity, radius of P one with respect to C. So we're essentially calling this center here C, the center here B, in the center here, A, and then we call the other radius r of P one with respect to B. So again, this is the radius of C. And this is the radius of wheel B that gets us to this point P one. And since we determine or the question tells us that there's no slip at p one, we know that the two velocities will have to be equal, so same magnitude and same direction. And so we can solve for the two, these two velocities from both sides. So from wheel C, and from we wheel B and then we equate them. So the two equations are as follows V p one, so this is the velocity at point one is going to be equal to omega c cross r of P one with respect to c. And v of P one is also equal to omega b cross r p one with respect to b. So, both of these equations need to be satisfied. So, instead of solving for VP one, which we don't really need in this problem, we can just equate these two cross products. And this equation. So basically, equating this to this gives us a relationship for the two angular velocities with respect to the size or the sizes of the two wheels. So, let's go ahead and do that. So let's plug in these vectors and extract just the magnitudes so that we can just solve for the magnitude. So for the first one using the first equation, so number one, V p one is equal to negative magnitude of omega c in the k hat direction. So again, this here, just to simplify is omega c, I just rewrote it in terms of the magnitude and the unit vector, which is negative k hat, cross product to negative 0.4 meters and the i hat, right, this negative 0.4 meters and the i hat is this radius over here, it is pointing in the backwards direction, so that's why it is negative. And if we do this cross product, we get the following 0.4 times the magnitude of omega c, j hat direction. So you can see that it's convenient for me to just put the magnitude here and the sign out here, so that I can just plug in the magnitude that we have just found into this equation without having to worry about positives or negatives, and getting rid of that cross product, right. So this is number one, let's do number two. V p one is going to be equal to omega B. In this case, I'll leave it as a vector cross product to 1.2 meters in the i hat direction. Now, this is a positive 1.2 meters because this radius is in the positive x hat direction based on how we defined it up there. And we see that this is going to be equal to 1.2 omega b in the j hat direction. Okay. So again, these two expressions stays the same, say the same thing. What I went ahead and did here is since omega v is in the positive direction, because rotation is defined positive in that direction, I just replaced it. So this here is equal to omega b magnitude in the k hat direction, right? And then I just pulled out the magnitude over there. I'm doing everything in terms of magnitudes so that I can get rid of these cross products and just get equations where I can just solve for these magnitudes. And it's important to note that both of these should be in the j hat direction. If they are not, then we are having we did something wrong there was an issue because we said that these two velocities need to match. So the directions need to match and now we need to also match their magnitudes right. So the magnitudes are going to be equated as follows 0.4 omega c is equal to 1.2 omega b. And from this, we get the following magnitude of omega b is equal to 0.4 meters divided by 1.2 meters times five radians per second, which is equal to 1.68 radians per second. And this is our magnitude of omega b. So, what we have essentially done is from omega c we derived omega B, now, we know omega b, we can derive omega a, and this is done it with the same exact method. So, similarly, the velocity of point two is going to be equal to omega b cross r p with p two, respect to B. And the other equation is V P two is equal to omega a, which is what we're trying to find cross two R P two with respect to a, these are all vectors. Right? So, it's the same format of equations we have above here. And we apply them once again, down here. Now, then we equate VP two and V the to VP two, so, we equate these two expressions, these two cross products. And given these two radii, we can determine omega A based on omega b we, which we just found, right, so, I'm not going to go through the same process because you get to the same relationship where, the this relationship over here, where the radius r of P two with respect to b. Again, this is the magnitude multiplied by the magnitude of omega b is equal to our P two with respect to A multiplied by the magnitude of omega A. Okay, again, the vertical lines mean the magnitude, so we have the same relationship, we know these dimensions, we also just found omega b, so we can solve directly for omega A. So, the magnitude of omega A is equal to omega b times r p two with respect to b divided by r of P two with respect to a. And these are again the magnitudes. So this is equal to 1.68 radians per second times 1.2 meters divided by 0.3 meters, which is equal to 6.67 radians per second. So, our final answer is omega A is equal to 6.67 radians per second. And this is going to be in the negative k hat direction. And this is our final answer. And if we go back, we can confirm that this is indeed in the negative k hat direction, whereas, this would be positive. So, again just to recap, we have derived these expressions vectorially. But this expression here always applies when we have two gears or in this case two wheels that involve no slipping So, the velocity at this point of contact is equal between the two. This expression always applies and we can use it to jump from wheel to wheel and determine the final angular velocity